Tools for Analysis of Accelerated Life and Degradation Test Data

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Tools and Education of Accelerated Testing

- Risk and reliability tools and education are becoming more effective and realistic due to reliance upon the **Physics of Failure (PoF)** approach
- PoF modeling uses **accelerated life** or **accelerated degradation testing** to assess model parameters based on the relations of stress, damage, and life
- Standards of Accelerated Testing Education
  - Conceptual definitions and methodologies
  - Supplementary example problems
  - Algorithms and codes

Overview of ALT/ADT Tools and Education

Part I

- Probabilistic physics of failure (PPoF)
- Mechanistic models of failure mechanisms
  - ❑ Stress-strength
  - ❑ Damage endurance
  - ❑ Performance-requirement
- Model examples
  - ❑ Fatigue
  - ❑ Wear
  - ❑ Corrosion
  - ❑ Creep

Overview of ALT/ADT Tools and Education

Part II

- Life-Stress Models
- Accelerated Life Testing (ALT) [2,3] Analysis:
  - Plotting Method
  - MLE
  - Bayesian Estimation
- Accelerated Degradation Testing (ADT)
- Step-Stress and Stress Varying Test
- Types of ALT and Pitfalls
- Test planning, Performance, Data Sorting, and Evaluation

Overview of ALT/ADT Tools and Education

Part III

- Applications of Physics-Based Models for:
  - ❑ Reliability Engineering
  - ❑ Prognosis Health Management (PHM)
- Comprehensive Case studies
- Field Examples

MLE ALT Example [1]

- Consider the following times-to-failure at three different stress levels

Stress (Temperature)	406 °K	436 °K	466 °K
Recorded Failure Times (Hours)	248	164	92
	456	176	105
	528	289	155
	731	319	184
	813	386	219
	965	459	235
	972	-	-
	1528	-	-

MLE ALT Example (Cont.)

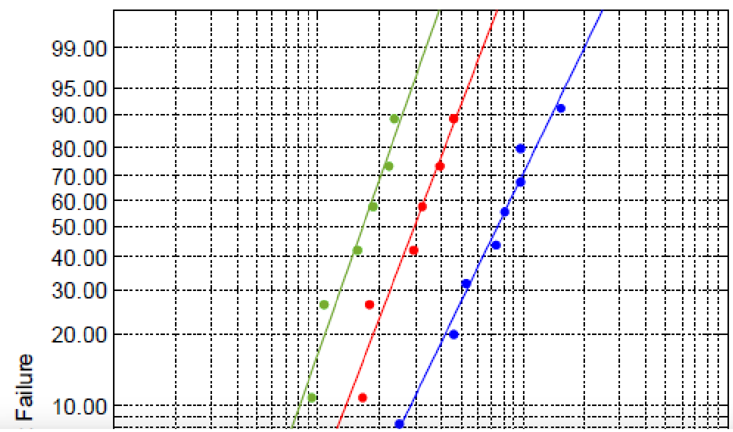
- If the data is best represented by a Weibull distribution and the Life (L)-Stress (T) model is best modeled as an Arrhenius model,

$$L = A\exp\frac{E_a}{KT}$$

  - A – Constant to be determined
  - E_a – Activation energy (eV)
  - T – temperature in Kelvin
  - K – Boltzmann constant (1/K=11605 eV)
- Estimate product life at given use-life 353 °K using the plotting method and the MLE method and compare

MLE ALT Example (Cont.)

- Plotting Method
  - **STEP 1**: Plot the life distribution at each stress level



For T=406K, n=8

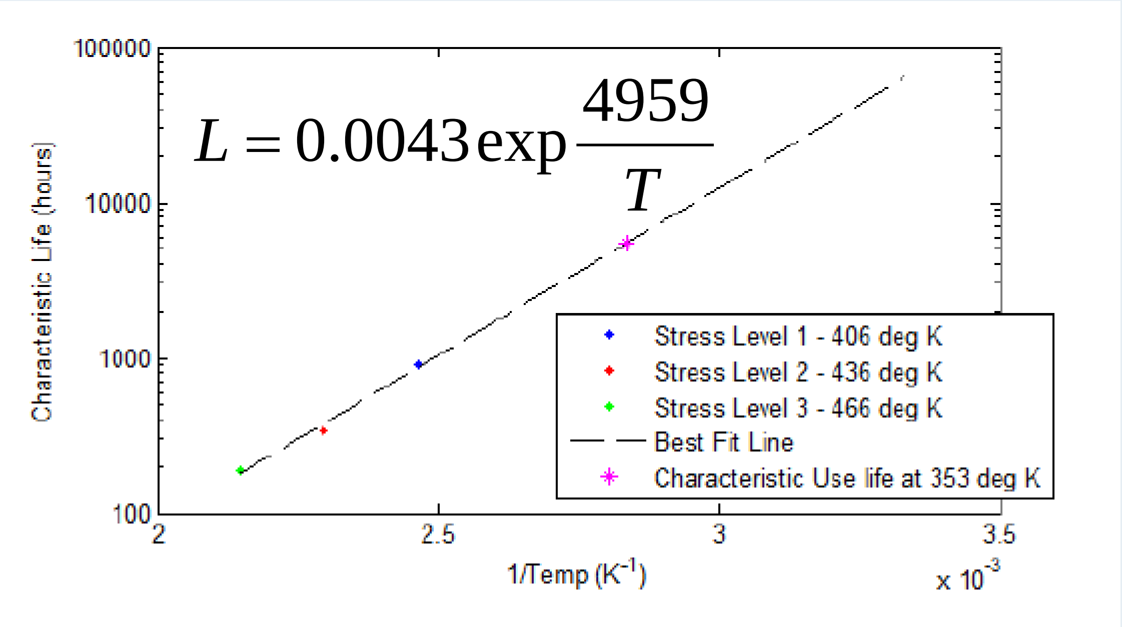
For T=466K, n=6

i	*ti*	*F={(i-0.375)/(n+0.25)}*100*
1	92	10.00
2	105	26.00
3	155	42.00
4	184	58.00
5	219	74.00
6	235	90.00

MLE ALT Example (Cont.)

- Plotting Method (Cont.)
  - **STEP 3**: Plot the life-stress at 63.2% failure and solve for Arrhenius parameters

$$L_{63.2\%} = A_{63.2\%} \exp \frac{E_{a_{63.2\%}}}{KT}$$



$$\hat{\beta} = (1.9 + 2.5 + 2.7)/3 = 2.5$$

$$\hat{A}_{63.2\%} = 0.0043; \hat{E}_{a_{63.2\%}} / K = 4959°K; \hat{E}_{a_{63.2\%}} = 0.43(eV)$$

$$L_{use} = 5{,}404 hours$$

MLE ALT Example (Cont.)

- MLE Method
  - **STEP 1**: Obtain the Weibull-Arrhenius likelihood expression

$$\Lambda = \sum_{i=1}^{M} N_i \ln\left\{ \frac{\beta}{A\exp\frac{E_a}{KT_i}} \left(\frac{t_i}{A\exp\frac{E_a}{KT_i}} \right)^{\beta-1} \exp\left[-\left(\frac{t_i}{A\exp\frac{E_a}{KT_i}} \right)^{\beta-1} \right] \right\}$$

  - **STEP 2**: Set the first derivatives of the likelihood w.r.t. the parameters to zero and solve simultaneously

$$\frac{\partial\Lambda}{\partial\beta} = 0; \frac{\partial\Lambda}{\partial A} = 0; \frac{\partial\Lambda}{\partial E_a} = 0$$

$$L = 0.0024\exp\frac{5201}{T}$$

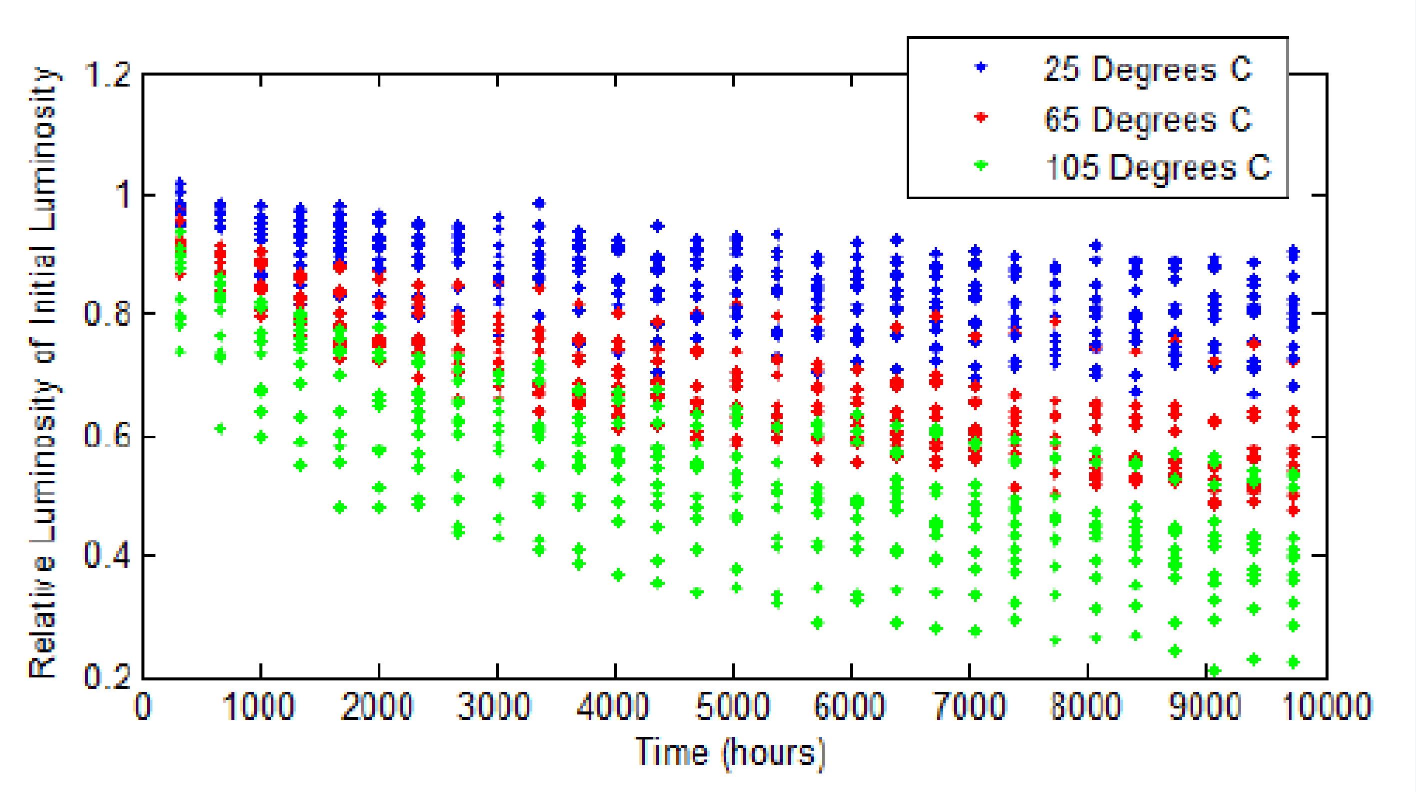
$$\hat{\beta} = 2.7; \hat{A} = 0.0024;$$

$$\hat{E}_a / K = 5201°K; \hat{E}_a = 0.45(eV)$$

$$L_{use} = 11{,}272 hours$$

Bayesian ADT Example [1, 4]

- Consider the following ADT where 25 LED units each are tested at temperatures 25 °C, 65 °C, and 105 °C [4]



Bayesian ADT Example (Cont.)

- LED degradation is modeled,

$$Y_{ijk} = \left\langle 1 + \beta_1 \left\{ t_{ijk} \exp\left[\beta_3 11605 \left(\frac{1}{T_u + 273.15} - \frac{1}{T_i + 273.15} \right) \right] \right\}^{\beta_2} \right\rangle^{-1} + \varepsilon_{ijk}$$

  - t_{ijk} – the k[th] time of the j[th] LED at the i[th] temperature level
  - ε_{ijk} – normal distribution **NOR(0, σ_ε)**
- If prior knowledge of the parameters β_1, β_2, β_3, and σ_ε is limited to a uniform distribution **UNIF(0,100)** for each, and an Arrhenius relationship is assumed for the ADT, find the posterior distribution and mean-time-to-failure for the LED at use temperature (T_u) 20 °C. **NOTE: Failure occurs at 50% luminosity**

Bayesian ADT Example (Cont.)

- Bayes Rule Mathematics

$$\pi_1(\vec{\theta} \mid DATA) = \frac{l(DATA \mid \vec{\theta})\pi_0(\vec{\theta})}{\int l(DATA \mid \vec{\theta})\pi_0(\vec{\theta})d\vec{\theta}}$$

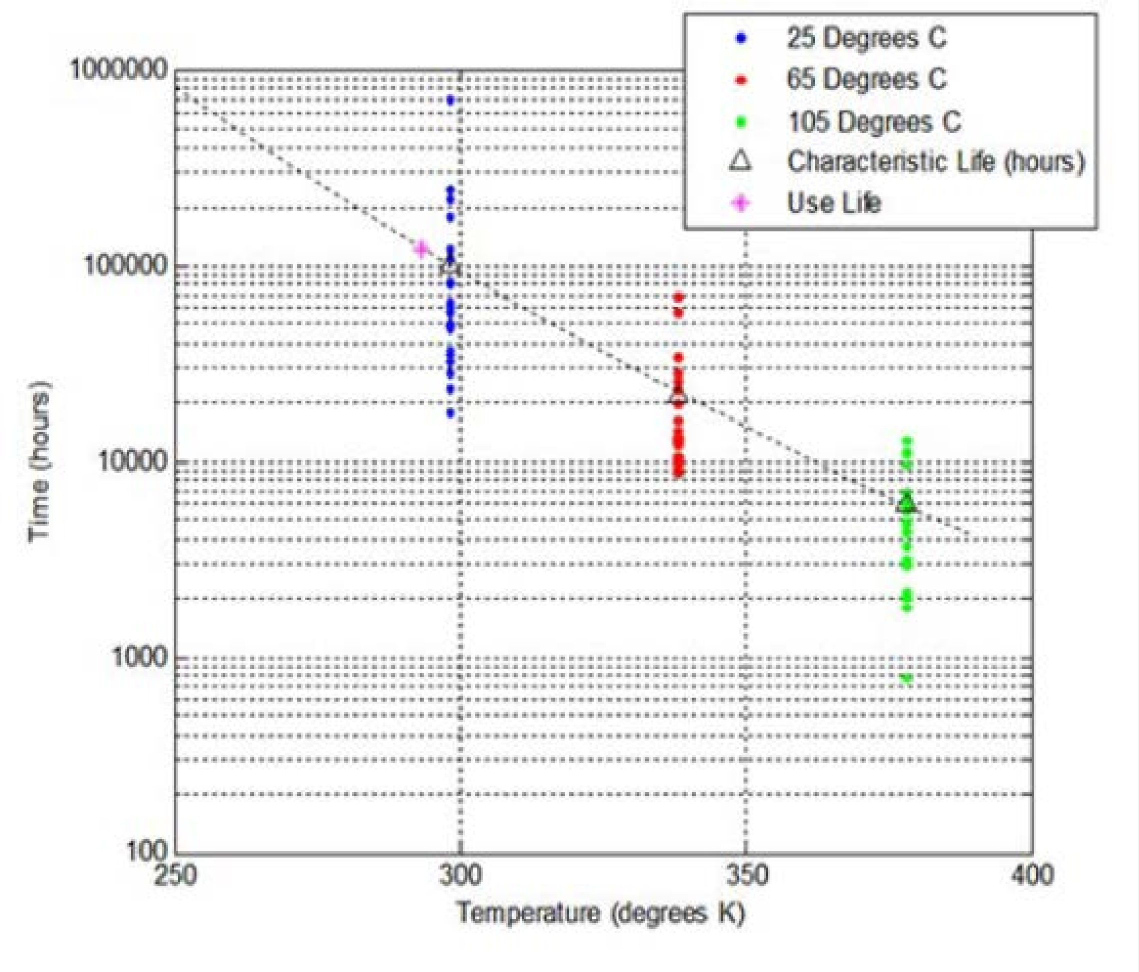
  - $\vec{\theta}$ – The model parameters vector, $[\theta_1 \quad \theta_2 \quad \cdots \quad \theta_n]$
  - $\pi_0(\vec{\theta})$ – prior distribution of parameters
  - $l(DATA \mid \vec{\theta})$ – likelihood
  - $\pi_1(\vec{\theta} \mid DATA)$ – posterior distribution of parameters

Bayesian ADT Example (Cont.)

- The posterior distribution is usually estimated using advanced numerical simulations. For example by using MCMC (Markov Chain Monte Carlo) simulation made in the MATLAB software we get:

Posterior Parameter	Lower Bound	Mean	Upper Bound
β_1	9.7×10^{-6}	0.027	0.062
B_2	0.29	0.61	1.0
β_3	0.0047	0.57	1.3
σ_ε	0.0071	0.091	0.20

Bayesian ADT Example (Cont.)

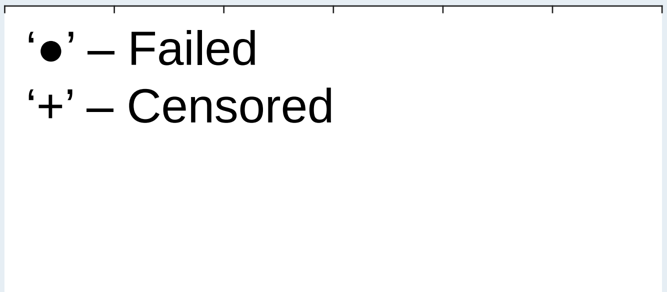


- The MTTF estimate is taken from the Bayesian posterior of each unit
- Recall failure occurs at 50% luminosity

$$MTTF_{use} = 121{,}975\,hours$$

Step-Stress Example [1, 5]

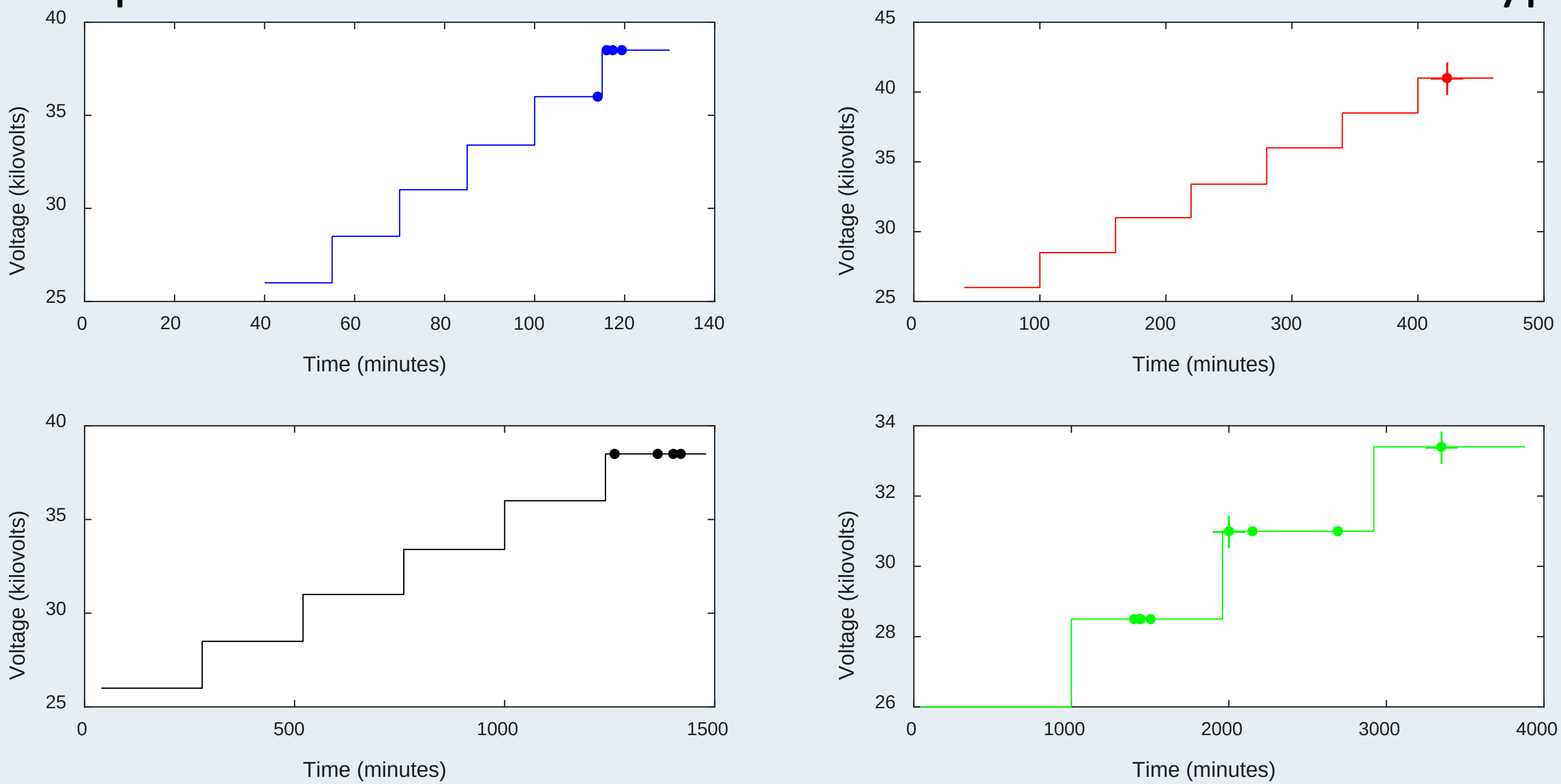
- Consider a step-stress test of cable insulation type 1:



Note: Stress is defined as voltage/cable thickness where nominal stress is 400 V/mm

Step-Stress Example (Cont.)

- If the prior data is obtained from this test of cable insulation type 2,



Update the model parameters using Bayesian estimation

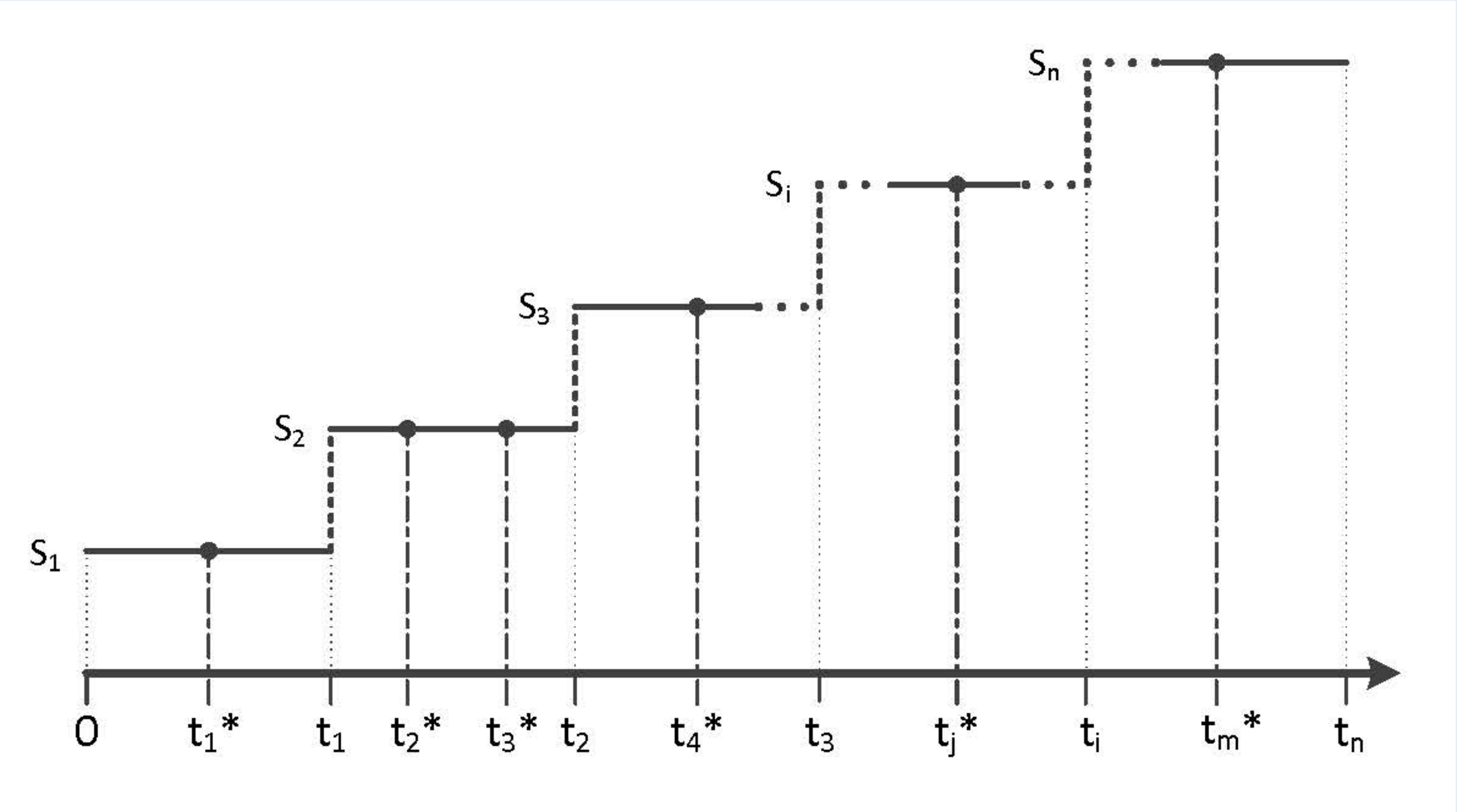
Step-Stress Example (Cont.)

- This time the life-stress model is represented by the inverse power law (IPL)

$$L = \frac{1}{AV^{p}}$$

  - A and p are constants to be determined
  - V is the stress in volts/mm

Step-Stress Example (Cont.)



Step-Stress Example (Cont.)

- The likelihood for an n-step-stress model is,

$$l = \prod_{i=1}^{n} \prod_{j=1}^{c_{ij}} f_i\left(t_{c_{i1}}^*, t_{c_{i2}}^*, \cdots, t_{c_{ij}}^*\right) \prod_{k=1}^{r_{ik}} \left[1 - F_i\left(t_{r_{i1}}^*, t_{r_{i2}}^*, \cdots, t_{r_{ik}}^*\right)\right]$$

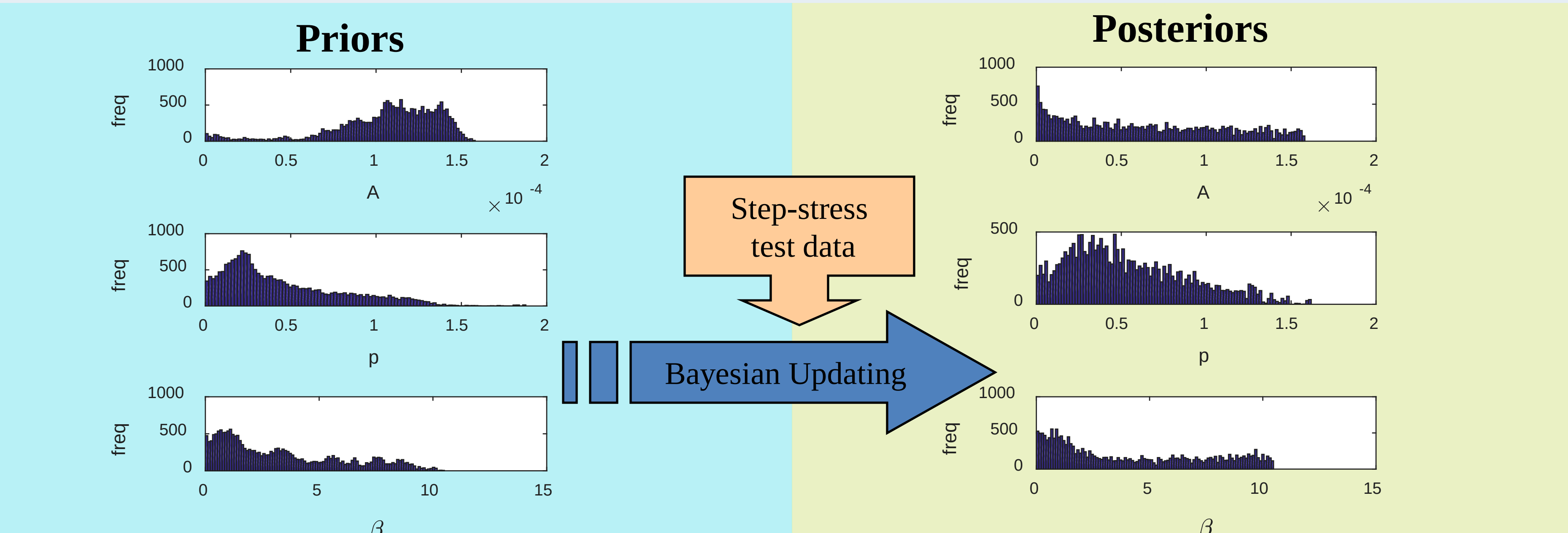
  - c_{ij} – number of failure data points at step i
  - $t_{c_{ij}}^*$ – time of j-th failure data point at step i
  - r_{ik} – number of right censored data points at step i
  - $t_{c_{ik}}^*$ – time of k-th right censored data point at step i, and

$$F_i\left(t_{c_{ij}}^*, V_i\right) = 1 - \exp\left\{-\left[\left(t_{c_{ij}}^* - t_{i-1} + \tau_{i-1}\right) A V_i^{\,p}\right]^{\beta}\right\}$$

$$f_i\left(t_{c_{ij}}^*, V_i\right) = \beta A V_i^{\,p} \left[\left(t_{c_{ij}}^* - t_{i-1} + \tau_{i-1}\right) A V_i^{\,p}\right]^{\beta-1} \exp\left\{-\left[\left(t_{c_{ij}}^* - t_{i-1} + \tau_{i-1}\right) A V_i^{\,p}\right]^{\beta}\right\}$$

$$\tau_{i-1} = \left(t_{i-1} - t_{i-2} + \tau_{i-2}\right)\left(\frac{V_{i-1}}{V_i}\right)^n$$

Step-Stress Example (Cont.)



Parameter	Lower Bound	Upper Bound	Parameter	Lower Bound	Upper Bound
A	4.38×10^{-10}	5.69×10^{-5}	A	3.70×10^{-9}	5.68×10^{-5}
p	1.37×10^{-4}	2.78	p	1.12×10^{-3}	1.82
β	1.74×10^{-4}	12.10	β	4.68×10^{-3}	12.10

Closing Remarks

- Reliability engineers should be cognizant and aware of the importance of accelerated testing practices
- Necessary components for understanding accelerated testing
  - Theory
  - Methods and models
  - Tools and applications

About Reuel Smith

- Reuel Smith is a PhD level Reliability Engineering graduate student at the A.J. Clark School of Engineering at the University of Maryland College Park. His current research is in the area of fatigue crack propagation, detection, and modeling.

- Reuel Smith received his M.S. degree in both Aerospace Engineering and Mechanical Engineering from the University of Maryland College Park.

About Dr. Mohammad Modarres

- Mohammad Modarres is the Nicole Y. Kim Eminent Professor of Engineering and Director Center for Risk and Reliability, A.J. Clark School of Engineering, University of Maryland, College Park. His research areas are probabilistic risk assessment and management, uncertainty analysis and physics of failure degradation modeling.

- Professor Modarres has over 350 papers in archival journals and proceedings of conferences, including several books in various areas of risk and reliability engineering. He is a University of Maryland Distinguished Scholar.

- Professor Modarres received his M.S. and PhD in Nuclear Engineering from MIT and M.S. in Mechanical Engineering also from MIT.

References

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Questions?

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Thank You

THE END!